

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATION

MATHEMATICS P1

2015

MARKS: 150

TIME: 3 hours

This question paper consists of 10 pages and 1 information sheet.



QUESTION 11.1 Solve for x :

1.1.1 $x(x-1)=0$ (2)

1.1.2 $2x^2 - 4x - 5 = 0$ (correct to TWO decimal places) (3)

1.1.3 $5^x = \frac{1}{125}$ (2)

1.1.4 $(x-3)(2-x) > 0$ (3)

1.2 Given: $f(x) = x + 1$ and $g(x) = \frac{-4}{x-3}$ 1.2.1 For which values of x will $g(x)$ be undefined? (1)1.2.2 Solve for x if $f(x) = g(x)$. (4)1.2.3 State whether the graph of f is a tangent to the graph of g when $f(x) = g(x)$. Motivate your answer. (2)

1.3 The distance between Joe's house and the supermarket is x km. He drives from his house to the supermarket at an average speed of y km/h. From the supermarket Joe returns to his house, along the same route, at an average speed that is one and a half times faster than the original average speed of y km/h. Calculate the overall average speed at which Joe drove from his house to the supermarket and back. Leave your answer in terms of y .

(6)
[23]**QUESTION 2**

The first three terms of an arithmetic sequence are 4; 13 and 22.

2.1 Write down the fourth term of this sequence. (1)

2.2 Determine the general term of the sequence. (2)

2.3 Consider the terms of this sequence which are even.
Calculate the sum of the first 25 terms which are even. (4)2.4 The original sequence (4; 13 and 22) forms the first differences of a new sequence with a first term equal to -6 . Determine a formula for the n^{th} term of this new sequence. (4)
[11]

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.



QUESTION 3

3.1 Given: $\sum_{p=4}^{21} (-3)^p$

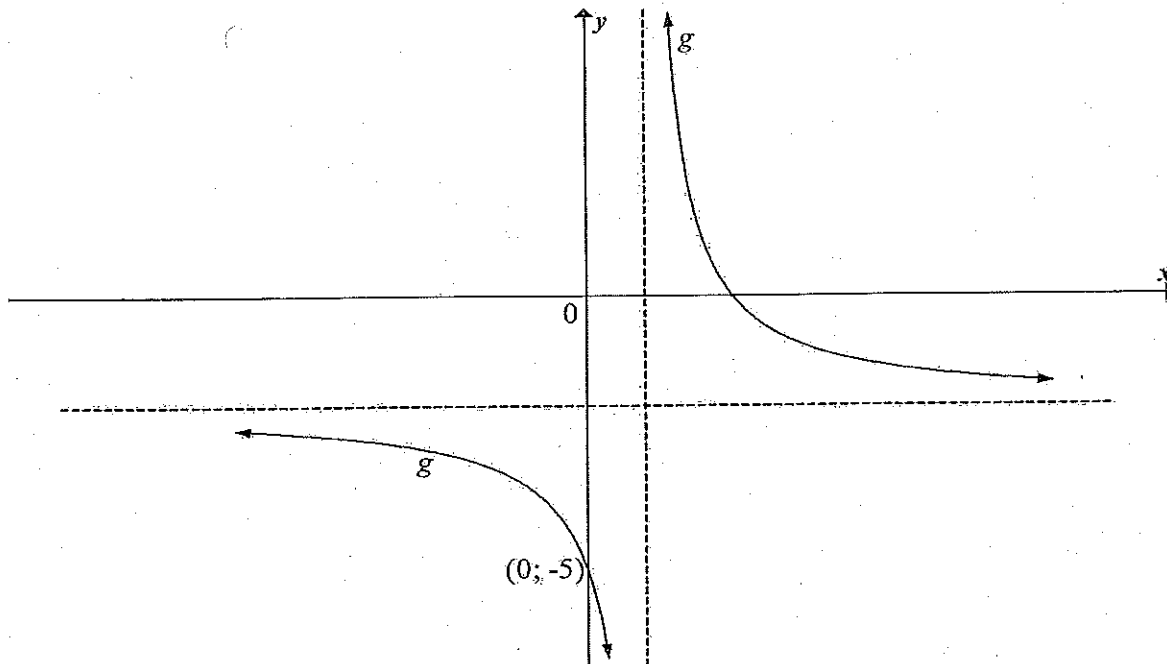
3.1.1 Write down the values of the first three terms of the series. (2)

3.1.2 Write down the value of the constant ratio. (1)

3.1.3 Will $\sum_{p=4}^{\infty} (-3)^p$ converge? Explain your answer. (2)3.1.4 Calculate $\sum_{p=4}^{21} (-3)^p x$. Give your answer in terms of x . (3)3.2 $6 - x$, 5 and $\sqrt{4x + 12}$ are the first three terms of an arithmetic sequence.3.2.1 Determine the value of x . (5)3.2.2 Calculate the value of the 10th term of this arithmetic sequence. (3)**[16]**

QUESTION 4

The diagram below shows the graph of $g(x) = \frac{a}{x-1} - 2$. The point $(0; -5)$ lies on g .



- 4.1 Write down the range of g . (2)
- 4.2 Determine the value of a . (2)
- 4.3 If another function h is defined as $h(x) = g(x-3) + 7$, determine the coordinates of the point of intersection of the asymptotes of h . (3)
- [7]



QUESTION 5

Given: $f(x) = \left(\frac{1}{4}\right)^x$

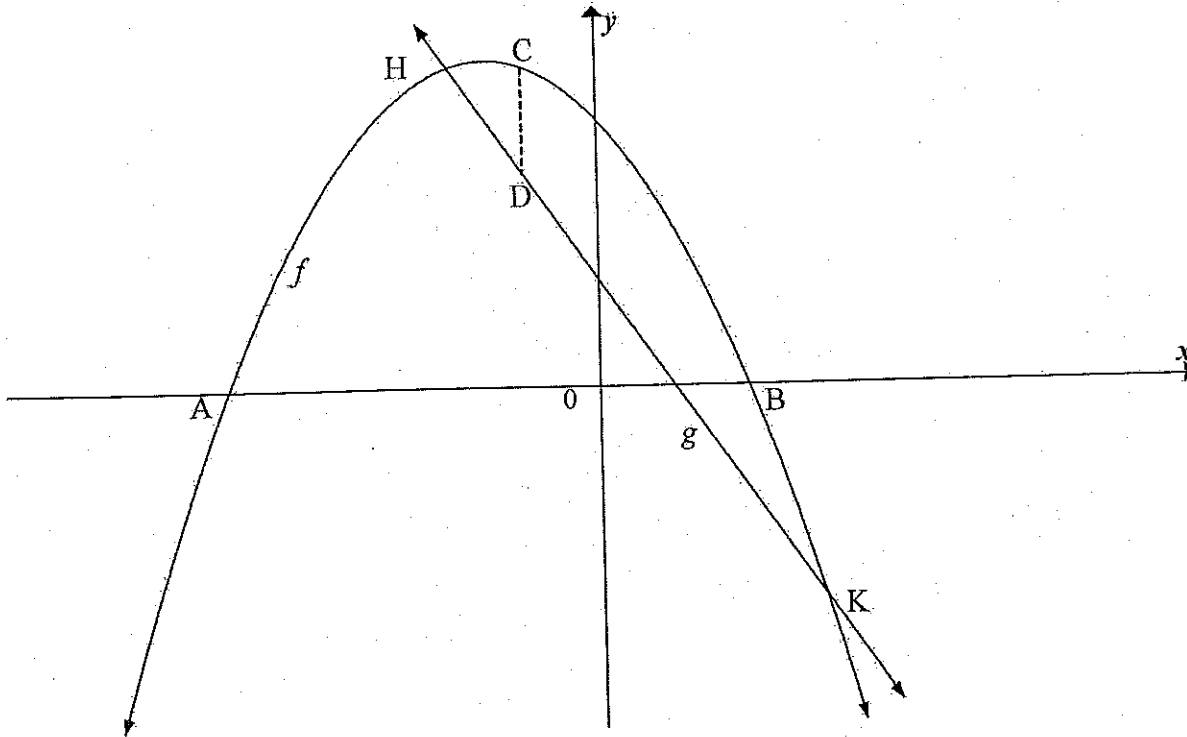
- 5.1 Determine the value of $f(-2)$. (2)
- 5.2 Write down the equation of $f^{-1}(x)$ in the form $y = \dots$ (2)
- 5.3 Andrew has no idea how to draw the graph of f^{-1} . Explain to Andrew how he may use the graph of f to draw the graph of f^{-1} . (1)
- 5.4 Hence or otherwise, sketch the graph of f^{-1} in your ANSWER BOOK. Clearly indicate ALL intercepts with the axes. (2)
- 5.5 Write down the domain of f^{-1} . (1)
- 5.6 For which values of x will $f^{-1}(x) \geq -2$? (2)
- 5.7 Given: $q = \log_{\frac{1}{4}} \frac{1}{2}$
- 5.7.1 Determine the value of q . (1)
- 5.7.2 Hence or otherwise, determine the coordinates of the point of intersection of f and f^{-1} . (3)

[14]



QUESTION 6

The sketch below shows the graphs of $g(x) = -12x + 12$ and $f(x) = -3x^2 - 9x + 30$. A and B are the x -intercepts of f and C is a point on f . D is a point on g such that CD is parallel to the y -axis. H and K are the points of intersection of f and g .



- 6.1 Determine the length of AB. (4)
 - 6.2 Determine the coordinates of K. (5)
 - 6.3 Determine the values of x for which $f(x) - g(x) \leq 0$. (3)
 - 6.4 Determine the maximum length of CD for $-2 \leq x \leq 3$. (5)
- [17]**



QUESTION 7

7.1 Anisha and Lindiwe each received R12 000 to invest for a period of 5 years. They invested the money at the same time according to the following options:

- Anisha: 8,5% p.a. simple interest. At the end of the 5 years she will receive an additional bonus pay-out of exactly 7,5% of the original amount invested.
- Lindiwe: 8,5% p.a. compounded quarterly.

Who will have the larger final amount after 5 years? Justify your answer with appropriate calculations. (6)

7.2 A company bought office furniture that cost R120 000. After how many years will the furniture depreciate to a value of R41 611,57 according to the reducing-balance method, if the rate of depreciation is 12,4% p.a.? (4)

7.3 Tebogo opened a savings account with a single deposit of R5 000 at the beginning of June 2015. He then made 24 monthly deposits of R800 at the end of every month, starting at the end of June 2015. The account earned interest at 15% p.a. compounded monthly.

Calculate the amount that should be in his savings account immediately after he makes the last deposit. (5)
[15]

QUESTION 8

8.1 If $f(x) = \frac{4}{x}$, determine $f'(x)$ from first principles. (5)

8.2 Determine:

8.2.1 $\frac{dy}{dx}$ if $y = 5x^2 + 5x + 2$ (2)

8.2.2 $D_x \left[\sqrt[3]{x^2} - \frac{1}{2}x \right]$ (3)

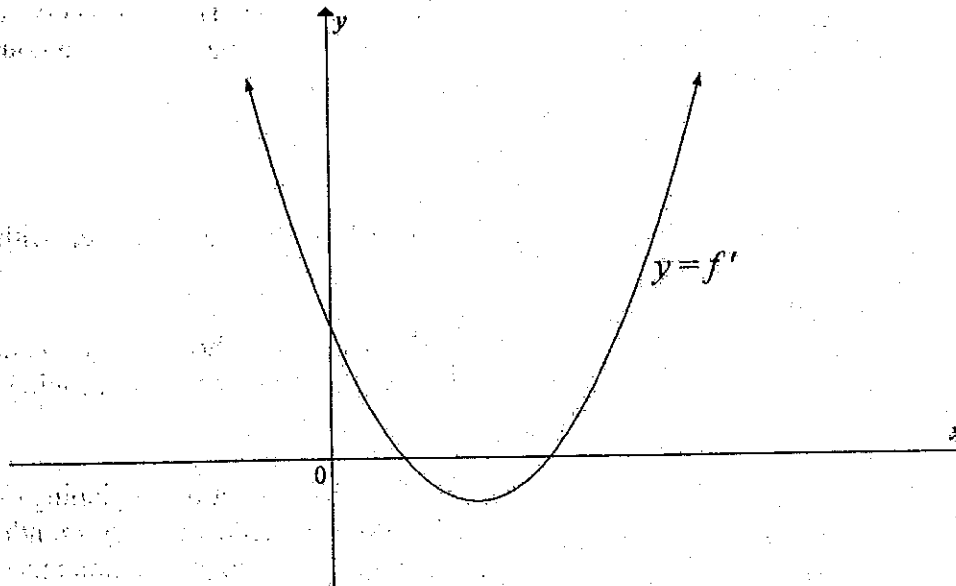
8.3 Given: $p(x) = x^3 + 2x$

Show, using relevant calculations, why it is not possible for a tangent drawn to the graph of p to have a negative gradient. (3)
[13]



QUESTION 9

The graph of $y = ax^2 + bx + c$ below represents the derivative of f .
It is given that $f'(1) = 0$, $f'(3) = 0$ and $f'(0) = 6$.



- 9.1 Write down the x -coordinates of the stationary points of f . (2)
- 9.2 For which value(s) of x is f strictly decreasing? (2)
- 9.3 Explain at which value of x the stationary point of f will be a local minimum. (2)
- 9.4 Determine the x -coordinate of the point of inflection of f . (1)
- 9.5 For which value(s) of x is f concave up? (2)

[9]

QUESTION 10

The mass of a baby in the first 30 days of life is given by

$$M(t) = t^3 - 9t^2 + 3\,000 \quad ; \quad 0 \leq t \leq 30.$$

t is the time in days and M is the mass of the baby in grams.

- 10.1 Write down the mass of the baby at birth. (1)
- 10.2 A baby's mass usually decreases in the first few days after birth.
On which day will the baby's mass return to its birth mass? (4)
- 10.3 On which day will this baby have a minimum mass? (4)
- 10.4 On which day will the baby's mass be decreasing the fastest? (2)

[11]



QUESTION 11

11.1 Zebra High School offers only two sporting activities, namely rugby and hockey.

The following information is given:

- There are 600 learners in the school.
- 372 learners play hockey.
- 288 learners play rugby.
- 56 of the learners play NO sport.
- The number of learners that play both hockey and rugby is x .

11.1.1 Represent the given information in a Venn diagram, in terms of x . (3)

11.1.2 Calculate the value of x . (2)

11.1.3 Are the events playing rugby and playing hockey mutually exclusive? Justify your answer. (2)

11.2 Another school, Tulani High, has a sports awards ceremony. Tulani High has a basketball team consisting of 5 players and a volleyball team consisting of 6 players.

11.2.1 All the basketball players sit in a single row at the ceremony. There are no restrictions on who sits in which position. In how many different ways can they be seated? (1)

11.2.2 The decision is taken that the captain must sit in the first seat of the row. The two vice-captains have to be seated next to each other in any of the remaining seats. In how many different ways can the basketball players be seated now? (3)

11.2.3 After the interval, the basketball team and the volleyball team sit in the same row at the ceremony. Calculate the probability that the basketball players will sit together and the volleyball players will sit together. Assume that seating positions are allocated randomly. Give your answer as a simplified fraction. (3)

[14]

TOTAL: 150



INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r};$$

$$-1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

